



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

TECHNICAL MATHEMATICS P1

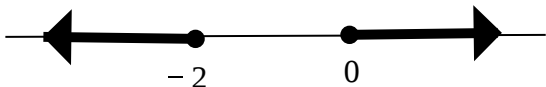
MARKING GUIDELINES

EXEMPLAR 2018

MARKS: 150

These marking guidelines consist of 12 pages.

QUESTION 1

1.1.1	$x(x+2) = 0$ $\therefore x = 0 \text{ or } x = -2$	$\checkmark x = 0$ $\checkmark x = -2$ (2)
1.1.2	$x(x+2) \geq 0$ $\therefore x \leq -2 \text{ OR } x \geq 0$ 	$\checkmark x \leq -2$ $\checkmark x \geq 0$ $\checkmark \text{ OR}$ $\checkmark$ Graphical representation (4)
1.2	$5x^2 = 2 + x$ $5x^2 - x - 2 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(5)(-2)}}{2(5)} = \frac{1 \pm \sqrt{41}}{10}$ $\therefore x \approx 0,74 \text{ or } x \approx -0,54$	$\checkmark$ Standard form $\checkmark$ Substitution into the quadratic formula $\checkmark x \approx 0,74$ $\checkmark x \approx -0,54$ (4)
1.3	$m - t - 1 = 0$ $m = t + 1$ $m^2 + t^2 = 5$ $(t+1)^2 + t^2 = 5$ $t^2 + 2t + 1 + t^2 - 5 = 0$ $2t^2 + 2t - 4 = 0$ $t^2 + t - 2 = 0$ $(t+2)(t-1) = 0$ $\therefore t = -2 \text{ or } t = 1$ $m = -2 + 1 = -1 \text{ or } m = 1 + 1 = 2$ OR $m - t - 1 = 0$ $t = m - 1$ $m^2 + t^2 = 9$ $m^2 + (m-1)^2 = 5$ $m^2 + m^2 - 2m + 1 - 5 = 0$ $2m^2 - 2m - 4 = 0$ $m^2 - m - 2 = 0$ $(m-2)(m+1) = 0$ $\therefore m = 2 \text{ or } m = -1$ $t = 2 - 1 = 1 \text{ or } t = -1 - 1 = -2$	$\checkmark$ Making m the subject $\checkmark$ Substitution $\checkmark$ Simplification $\checkmark$ Factors $\checkmark$ Both values of t $\checkmark$ Both values of m OR $\checkmark$ Making t the subject $\checkmark$ Substitution $\checkmark$ Simplification $\checkmark$ Factors $\checkmark$ Both values of m $\checkmark$ Both values of t (6)

1.4.1	$\varepsilon = \frac{L_2 - L_1}{L_1}$ $\varepsilon L_1 = L_2 - L_1$ $\varepsilon L_1 + L_1 = L_2$ $L_1(\varepsilon + 1) = L_2$ $L_1 = \frac{L_2}{(\varepsilon + 1)}$	$\varepsilon = \frac{L_2}{L_1} - 1$ $\varepsilon + 1 = \frac{L_2}{L_1}$ $L_1(\varepsilon + 1) = L_2$ $\therefore L_1 = \frac{L_2}{(\varepsilon + 1)}$	✓ multiply with LCD ✓ common factor ✓ divide by factor (3)
1.4.2	$L_1 = \frac{L_2}{\varepsilon + 1}$ $= \frac{18}{1 + 0,8} \text{ cm}$ $= 10 \text{ cm}$		✓ Substitution ✓ Simplification (2)
1.4.3	$10 = 8 + 2 = 2^3 + 2$ $= 1010_2$		✓ $2^3 + 2$ ✓ 1010_2 , (2)
1.5	$12 \times 0,00361$ $= 0,04332$ $= 4,332 \times 10^{-2}$		✓ 0,04332 ✓ $4,332 \times 10^{-2}$ [25]

QUESTION 2

2.1.1	$p = -1$	✓ $p = -1$ (1)
2.1.2	$9 - 3p < 0$ $9 < 3p$ $\therefore p > 3$	✓ $9 - 3p < 0$ ✓ $p > 3$ (2)
2.1.3	0 OR 3	✓ 0 OR 3 (1)
2.2	$x^2 - 4x + (k - 1) = 0$ For equal roots, $\Delta = b^2 - 4ac = 0$ $(-4)^2 - 4(1)(k - 1) = 0$ $16 - 4k + 4 = 0$ $-4k = -20$ $\therefore k = 5$	✓ For equal roots, $\Delta = 0$ ✓ Substitution ✓ Simplification ✓ Value of k (4) [8]

QUESTION 3

3.1.1	$\frac{5 \times 2^{n-1} - 2^n}{2^n}$ $= \frac{2^n (5 \times 2^{-1} - 1)}{2^n}$ $= 5 \times \frac{1}{2} - 1 = \frac{3}{2}$ OR $\frac{5 \times 2^{n-1} - 2^n}{2^n}$ $= \frac{5 \times 2^{n-1}}{2^n} - \frac{2^n}{2^n} = 5 \times 2^{-1} - 1$ $= 2 \frac{1}{2} - 1 = \frac{3}{2}$	✓✓ Common factor ✓ Simplification ✓✓ Dividing each term by the denominator ✓ Simplification (3)
3.1.2	$\sqrt{64+16} - \sqrt{20}$ $= \sqrt{80} - \sqrt{4 \times 5}$ $= 4\sqrt{5} - 2\sqrt{5}$ $= 2\sqrt{5}$	✓ Addition ✓ Simplified surd ✓ Simplified surd ✓ Simplification (4)
3.1.3	$\log_6 216 \times \log 0,001$ $= \log_6 6^3 \times \log \frac{1}{1000}$ $= \log_6 6^3 \times \log 10^{-3}$ $= 3 \log_6 6 \times (-3 \log 10)$ $= 3(1) \times (-3)(1)$ $= -9$	✓ $\log_6 6^3$ ✓ $\log 10^{-3}$ ✓ $3 \log_6 6 - 3 \log 10$ ✓ Simplification (4)
3.2.2	$\log(x+18) - \log x = 1$ $\log \frac{(x+18)}{x} = 1$ $\frac{(x+18)}{x} = 10$ $10x = x+18$ $9x = 18$ $\therefore x = 2$	✓ Apply log property ✓ Change from log form to exp. Form ✓ Simplification ✓ Value of x (4)

3.3	$z = 3 + \sqrt{3}i$ $ z = r = \sqrt{x^2 + y^2}$ $= \sqrt{(3)^2 + (\sqrt{3})^2} = \sqrt{12}$ $\tan \theta = \frac{\sqrt{3}}{3}$ $\theta = 30^\circ$ $z = \sqrt{12} \operatorname{cis}(30^\circ) \quad \text{OR} \quad z = \sqrt{12} [\cos 30^\circ + i \sin 30^\circ]$	✓ Calculating the modulus ✓ Simplification ✓ $\tan \theta = \frac{\sqrt{3}}{3}$ ✓ Argument ✓ Correct polar form (5)
3.4	$x + yi = (3 + 5i)(2 - 7i)$ $x + yi = 6 - 11i - 35i^2$ $x + yi = 6 - 11i - 35(-1)$ $x + yi = 6 - 11i + 35$ $x + yi = 41 - 11i$ $\therefore x = 41 \text{ and } y = -11$	✓ $6 - 11i - 35i^2$ ✓ $i^2 = -1$ ✓ $x = 41$ ✓ $y = -11$ (4) [24]

QUESTION 4

4.1.1	x – intercepts, $f(x) = 0$ $2x^2 + 4x - 6 = 0$ $2(x+3)(x-1) = 0$ OR $(x+3)(2x-2) = 0$ $\therefore x = -3$ or $x = 1$ $\therefore B(1; 0)$	✓ Finding other factor ✓ Coordinates of B. (2)
4.1.2	$f(x) = 2x^2 + 4x - 6$ $\left(\frac{-b}{2a}; \frac{4ac - b^2}{4a} \right) = \left(\frac{-4}{2(2)}; \frac{4(2)(-6) - (4)^2}{4(2)} \right)$ $\therefore D(-1; -8)$ OR $x = \frac{-b}{2a} = \frac{-4}{2(2)}$ $\therefore x = -1$ $f(-1) = 2(-1)^2 + 4(-1) - 6 = -8$ $\therefore D(-1; -8)$ OR $x_D = \frac{-3+1}{2} = -1$ $f(-1) = 2(-1)^2 + 4(-1) - 6 = -8$ $\therefore D(-1; -8)$ OR $f(x) = 2x^2 + 4x - 6$ $f'(x) = 4x + 4 = 0$ $\therefore x = -1$ $f(-1) = 2(-1)^2 + 4(-1) - 6 = -8$ $\therefore D(-1; -8)$	✓✓ Substitution in formula ✓ Coordinates of D OR ✓ Substitution in formula ✓ Substitution to find y ✓ Coordinates of D OR ✓ Using x -intercepts ✓ Substitution to find y ✓ Coordinates of D OR ✓ Using the derivative ✓ Substitution to find y ✓ Coordinates of D (3)
4.1.3	$g(x) = k^x + q$ $10 = k^2 + 6$ $k^2 = 4$ $\therefore k = 2$	✓ Substituting coordinates of Q ✓ Simplified equation ✓ Correct value of k . (3)
4.1.4	$y = 6$	✓ $y = 6$ (1)
4.1.5	$-3 < x < 1$	✓ Correct critical values ✓ Correct notation (2)

4.2.1	$x = 0$ and $y = 1$	$\checkmark x = 0$ $\checkmark y = 1$ (2)
4.2.2	$h(x) = \frac{3}{x} + 1$ $0 = \frac{3}{x} + 1$ $-1 = \frac{3}{x}$ $\therefore x = -3$	$\checkmark$ Substituting coordinates of Q $\checkmark$ Value of x (2)
4.2.3	$r = 2$	$\checkmark r = 2$ (1)
4.2.4		$\checkmark$ Shape of h $\checkmark$ Asymptote $\checkmark$ x -intercept $\checkmark$ Any other point on the graph of h $\checkmark$ Shape of g $\checkmark$ x -intercepts of g $\checkmark$ y -intercept of g (7)
4.2.5	$0 \leq y \leq 2$	$\checkmark 0 \leq y$ $\checkmark y \leq 2$ (2) [25]

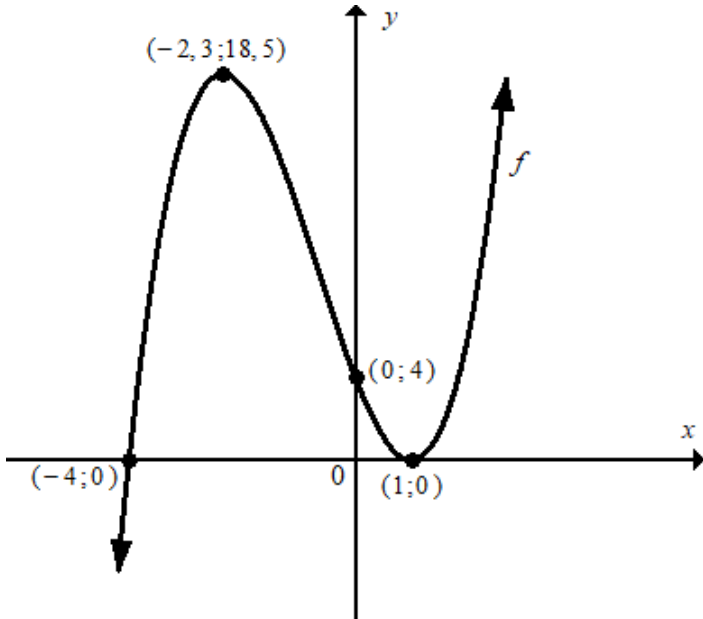
QUESTION 5

5.1	$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$ $i_{eff} = \left(1 + \frac{0,072}{2}\right)^2 - 1$ $\approx 0,073296$ $\therefore$ annual effective interest rate is 7,33%	✓ Correct substitution ✓ Simplification ✓ Effective rate as % (3)
5.2	$A = P(1 - i)^n$ $70 = 220(1 - 0,08)^n$ $\frac{7}{22} = (0,92)^n$ $n = \log_{0,92} \frac{7}{22}$ $\therefore n \approx 13,73363166$ It will take approximately 14 minutes.	✓ Correct formula ✓ Correct substitution ✓ Simplified power form ✓ Using logarithms ✓ Nearest minute (5)
5.2.2	Value of A after 3 years: $A = P(1 + i)^n$ $A = R150\,000 \left(1 + \frac{10,5\%}{4}\right)^{3 \times 4}$ $= R\,204\,705,40$ Value of P after withdrawal: $P = R\,204\,705,40 - R\,30\,000 = R\,174\,705,40$ Amount received at the end of the investment period: $A = R174\,705,40 \left(1 + \frac{10,5\%}{4}\right)^{2 \times 4}$ $\therefore A = R\,214\,947,15$	✓ Correct formula ✓ Correct substitution ✓ R204 705,40 ✓ P = R174 705,40 ✓ Correct substitution ✓ Final amount (6) [14]

QUESTION 6

6.1	$f(x) = 2x^2 - 3$ Average gradient = $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$ $= \frac{[2(1)^2 - 3] - [2(-2)^2 - 3]}{1 - (-2)}$ $= \frac{-1 - 5}{3}$ $= -2$	✓Corresponding y-value ✓Corresponding y-value ✓Substitution in formula ✓Simplification (4)
6.2	$f(x) = 4 - 3x$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{[4 - 3(x+h)] - (4 - 3x)}{h}$ $= \lim_{h \rightarrow 0} \frac{4 - 3x - 3h - 4 + 3x}{h}$ $= \lim_{h \rightarrow 0} \frac{-3h}{h}$ $= \lim_{h \rightarrow 0} (-3)$ $= -3$	✓Definition ✓Substitution in the definition ✓Simplification (removing brackets) ✓Simplification (division) ✓Simplification (5)
6.3	$y = \frac{2}{x^3} + \sqrt{x}$ $y = 2x^{-3} + x^{\frac{1}{2}}$ $\frac{dy}{dx} = -6x^{-4} + \frac{1}{2}x^{-\frac{1}{2}}$	✓ $2x^{-3}$ ✓ $x^{\frac{1}{2}}$ ✓ $-6x^{-4}$ ✓ $\frac{1}{2}x^{-\frac{1}{2}}$ (4)
6.4	$g(x) = -x^2 - x$ $g(2) = -(2)^2 - 2 = -6$ The point of contact is (2; -6) $g'(x) = -2x - 1$ $\therefore m_{\tan} = g'(2) = -2(2) - 1 = -5$ $y = mx + c$ OR $y - y_1 = m(x - x_1)$ $-6 = -5(2) + c$ OR $y - (-6) = -5(x - 2)$ $c = 4$ OR $y + 6 = -5x + 10$ $\therefore y = -5x + 4$	✓ value of y ✓ $m_{\tan} = -5$ ✓Correct substitution ✓Value of c (simplification) ✓Equation (any form) (5) [18]

QUESTION 7

7.1	$f(x) = x^3 + 2x^2 - 7x + 4$ $f(1) = (1)^3 + 2(1)^2 - 7(1) + 4$ $\therefore f(1) = 0$ $\therefore x - 1$ is a factor of f	✓ Substitution ✓ 0 (2)
7.2	x -intercepts: $f(x) = 0$ $x^3 + 2x^2 - 7x + 4 = 0$ $(x - 1)(x^2 + 3x - 4) = 0$ $(x - 1)(x - 1)(x + 4) = 0$ $x = 1$ or $x = -4$	✓ $(x^2 + 3x - 4)$ (quadratic) ✓ $(x - 1)(x - 1)(x + 4)$ (linear) ✓ x -intercepts (3)
7.3	$f(x) = x^3 + 2x^2 - 7x + 4$ $f'(x) = 3x^2 + 4x - 7$ $f'(x) = 0$ $\therefore 3x^2 + 4x - 7 = 0$ $(3x + 7)(x - 1) = 0$ $\therefore x = -\frac{7}{3}$ or $x = 1$ $(-2, 3; 18, 5)$ and $(1; 0)$	✓ Derivative ✓ $f'(x) = 0$ ✓ Factorisation ✓ Both values of x ✓ Coordinates of the turnings (5)
7.4		✓ Shape ✓ Intercepts with x -axis ✓ y -intercept ✓ Turning points (4) [14]

QUESTION 8

8.1.1	After 2 hrs $D(2) = 4 + 0,5(2)^2 - 0,25(2)^3 \text{ m}$ $= 4\text{m}$	✓ Substituting 2 ✓ Simplification (2)
8.1.2	$D = 4 + 0,5t^2 - 0,25t^3$ $D'(t) = t - 0,75t^2$ At 12:00 (3 hours later): $D'(3) = (3) - 0,75(3)^2$ $= -3,75\text{m.h}^{-1}$ $\therefore$	✓ Derivative ✓ Substitution of 3 ✓ Simplified rate (3)
8.2.1	$P = -3v^2 + 30v$ Neither profit nor loss at $P = 0$ $-3v^2 + 30v = 0$ $-3v(v - 10) = 0$ $\therefore v = 0 \text{ or } v = 10$ $v = 10\text{km.h}^{-1}$	✓ $P = 0$ ✓ Factors ✓ Correct value of v (3)
8.2.2	$P = -3v^2 + 30v$ $\frac{dP}{dv} = -6v + 30 = 0$ $\therefore v = 5\text{km.h}^{-1}$	✓ Derivative ✓ Equating to 0 ✓ Value of v (3)
8.2.3	$P_{\max} (\text{in R1000}) = -3(5)^2 + 30(5) = 75$ OR R75 000	✓ Substitution ✓ Profit in R1 000 (2) [13]

QUESTION 9

9.1	$\int \left(x^{-4} + \frac{7}{x} - 1 \right) dx$ $= \int x^{-4} dx + 7 \int \frac{1}{x} dx - \int dx$ $= \frac{x^{-3}}{-3} + 7 \ln x - x + C$	$\checkmark \frac{x^{-5}}{-5}$ $\checkmark 7 \ln x$ $\checkmark -x$ $\checkmark C$ (4)
9.2	$h(x) = -2x^2 - 6x$ $\int_{-3}^0 (-2x^2 - 6x) dx$ $= \left[-\frac{2x^3}{3} - 3x^2 \right]_{-3}^0$ $= \left[\left(-\frac{2(0)^3}{3} - 3(0)^2 \right) - \left(-\frac{2(-3)^3}{3} - 3(-3)^2 \right) \right]$ $= -18 + 27$ $= 9 \text{ units square}$	$\checkmark -\frac{2x^3}{3}; \quad \checkmark -3x^2$ $\checkmark \text{Substituting } 0$ $\checkmark \text{Substituting } -3$ $\checkmark \text{Simplification}$ (5) [9]

TOTAL: 150