



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE
*NASIONALE
SENIOR SERTIFIKAAT***

GRADE/*GRAAD* 11

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2016

MEMORANDUM

MARKS/*PUNTE*: 150

**This memorandum consists of 20 pages.
*Hierdie memorandum bestaan uit 20 bladsye.***

NOTE:

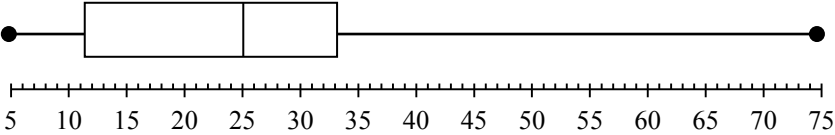
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking memorandum.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.
- Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing.
- Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.

QUESTION/VRAAG 1

5	8	15	20	25	27	31	36	75
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1.1	Range/Omvang = $75 - 5$ = 70	✓ answer/antw (1)
1.2	Std dev/Std afwyking = 19,56	✓ ✓ answer/antw (2)
1.3	Median/Mediaan = 25	✓ answer/antw (1)
1.4	$Q_1 = \frac{8+15}{2} = 11,5$ $Q_3 = \frac{31+36}{2} = 33,5$ $IQR = Q_3 - Q_1$ $= 33,5 - 11,5$ $= 22$	✓ $Q_1 = 11,5$ ✓ $Q_3 = 33,5$ ✓ answer/antw (3)
1.5		✓ box/mond ✓ whiskers/snor ✓ min and max (maks) (3)
1.6	Skewed to the right/skeef na regs Positively skewed/positief skeef	✓ answer/antw (1)
1.7	Outlier/uitskieter = 75 OR/OF $33.5 + 1.5(22) = 66.5$ Outlier/uitskieter = 75	✓ answer/antw ✓ answer/antw (1) [12]

QUESTION/VRAAG 2

2.1	<table border="1"> <thead> <tr> <th>TIME SPENT/ <i>TYD SPANDEER</i> (IN MINUTES/ <i>MINUTE</i>)</th><th>FREQUENCY <i>FREKWENSIE</i> f</th><th>CUMULATIVE FREQUENCY/ <i>KUMULATIEWE</i> <i>FREKWENSIE</i> f</th></tr> </thead> <tbody> <tr><td>$95 < x \leq 105$</td><td>15</td><td>15</td></tr> <tr><td>$105 < x \leq 115$</td><td>27</td><td>42</td></tr> <tr><td>$115 < x \leq 125$</td><td>43</td><td>85</td></tr> <tr><td>$125 < x \leq 135$</td><td>52</td><td>137</td></tr> <tr><td>$135 < x \leq 145$</td><td>28</td><td>165</td></tr> <tr><td>$145 < x \leq 155$</td><td>21</td><td>186</td></tr> <tr><td>$155 < x \leq 165$</td><td>10</td><td>196</td></tr> <tr><td>$165 < x \leq 175$</td><td>4</td><td>200</td></tr> </tbody> </table>	TIME SPENT/ <i>TYD SPANDEER</i> (IN MINUTES/ <i>MINUTE</i>)	FREQUENCY <i>FREKWENSIE</i> f	CUMULATIVE FREQUENCY/ <i>KUMULATIEWE</i> <i>FREKWENSIE</i> f	$95 < x \leq 105$	15	15	$105 < x \leq 115$	27	42	$115 < x \leq 125$	43	85	$125 < x \leq 135$	52	137	$135 < x \leq 145$	28	165	$145 < x \leq 155$	21	186	$155 < x \leq 165$	10	196	$165 < x \leq 175$	4	200	✓✓ correct CF values/ <i>korrekte KF-waardes</i> (2)
TIME SPENT/ <i>TYD SPANDEER</i> (IN MINUTES/ <i>MINUTE</i>)	FREQUENCY <i>FREKWENSIE</i> f	CUMULATIVE FREQUENCY/ <i>KUMULATIEWE</i> <i>FREKWENSIE</i> f																											
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2.2	OGIVE/OGIEF	✓✓✓ 8 points correct OR/OF ✓✓ 4–7 points correct OR/OF ✓ 1–3 points correct (3)																											
2.3	$Q_1 = 118$ Accept any answer between (115 and 120)	✓✓ answer/antw (2)																											
2.4	<i>Number of learners / Getal leerders</i> = $200 - 150$ = 50 Accept 150 or any other reading between (145 and 155)	✓ 150 ✓ 50 (2) [9]																											

QUESTION/VRAAG 3

3.1	$M = \left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2} \right)$ $= \left(\frac{6+2}{2} ; \frac{-2+15}{2} \right)$ $= \left(4 ; \frac{13}{2} \right)$	✓ subst into/in midpt form/ mdpnt vorm. ✓ answer/antw (2)
3.2	$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{15 - 3}{2 - (-4)}$ $= 2$ $m_{MN} = m_{BC} = 2 \quad [BC \parallel MN]$	✓ subst into gradient form./subst in gradiëntvorm ✓ answer/antw ✓ gradients equal/ gradiënte gelyk (3)
3.3	$y - y_1 = m(x - x_1) \qquad y = mx + c$ $y - \frac{13}{2} = 2(x - 4) \qquad \frac{13}{2} = 2(4) + c$ $y = 2x - \frac{3}{2} \qquad \textbf{OR/OF} \qquad -\frac{3}{2} = c$ $\qquad \qquad \qquad y = 2x - \frac{3}{2}$	✓ subst $\left(4 ; \frac{13}{2} \right)$ and $m = 2$ into str line eq. ✓ answer/antw (2)

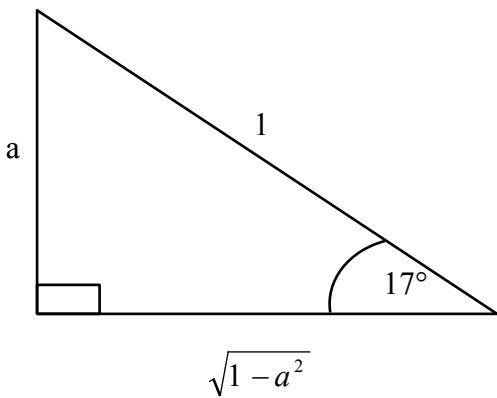
3.4	N is a midpoint of AC [Line through midpoint of one side parallel to second side] $N\left(\frac{-4+6}{2}; \frac{3+(-2)}{2}\right)$ $= N\left(1; \frac{1}{2}\right)$ OR/OF $m_{AC} = \frac{3-(-2)}{(-4)-6}$ $= -\frac{1}{2}$ Equation of AC $y - y_1 = m(x - x_1)$ $y - 3 = -\frac{1}{2}(x - (-4))$ $y = -\frac{1}{2}x + 1$ $-\frac{1}{2}x + 1 = 2x - \frac{3}{2}$ $-x + 2 = 4x - 3$ $x = 1$ $y = 2(1) - \frac{3}{2}$ $= \frac{1}{2}$ $N\left(1; \frac{1}{2}\right)$	✓ S ✓ R ✓ x-value/waarde ✓ y-value/waarde (4) ✓ gradient of AC ✓ equation of AC ✓ equating/gelykstelling ✓ $N\left(1; \frac{1}{2}\right)$ (4)
3.5	N is the midpoint of BD and the midpoint of AC [diagonals of parm bisect] <i>N is die midpt v BD en midpt v AC [hoeklyne van parm halveer]</i> $\left(\frac{2+x}{2}; \frac{y+15}{2}\right) = \left(1; \frac{1}{2}\right)$ $\frac{2+x}{2} = 1 \qquad \frac{y+15}{2} = \frac{1}{2}$ $x = 0 \qquad y = -14$ $D(0; -14)$	✓ $\frac{2+x}{2} = 1$ ✓ $\frac{y+15}{2} = \frac{1}{2}$ ✓ $x = 0$ ✓ $y = -14$ (4)

	OR/OF From B to A $(x; y) \rightarrow (x + 4; y - 17)$ $D(-4 + 4; 3 - 17)$ $D(0; -14)$	✓ $x + 4$ ✓ $y - 17$ ✓ subst ✓ $D(0; -14)$ (4) [15]
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QUESTION/VRAAG 4

4.1	$m_{MP} = m_{PN}$ $\frac{2-0}{0-k} = \frac{4-2}{3-0}$ $\frac{2}{-k} = \frac{2}{3}$ $k = -3$	✓ $m_{MP} = m_{PN}$ ✓ subst. into gradient form. ✓ answer/antw (3)
4.2	$\tan \alpha = m_{PN}$ $\tan \alpha = \frac{2}{3}$ $\alpha = 33,69^\circ$ $\tan \beta = m_{AB}$ $\tan \beta = -\frac{1}{2}$ $\beta = -26,57^\circ + 180^\circ$ $= 153,43^\circ$ $\theta = 153,43^\circ - 33,69^\circ$ $= 119,74^\circ$	✓ $\alpha = 33,69^\circ$ ✓ $\tan \beta = -\frac{1}{2}$ ✓ $153,43^\circ$ ✓ $119,74^\circ$ (4)
4.3	$-\frac{1}{2}x + 4 = 0$ $x = 8$ $R(8;0)$ $MR = 8 - (-3)$ $= 11 \text{ units}$	✓ $y = 0$ ✓ $R(8;0)$ ✓ answer/antw (3)
4.4	$\text{Area of } \triangle MNR = \frac{1}{2}(MR) \cdot \perp \text{ height}$ $= \frac{1}{2}(11)(y - \text{value of } N)$ $= \frac{1}{2}(11)(4)$ $= 22 \quad \text{sq units/vk eenh}$ OR/OF	✓ area formula ✓ subst y-value of N ✓ answer/antw

QUESTION/VRAAG 5

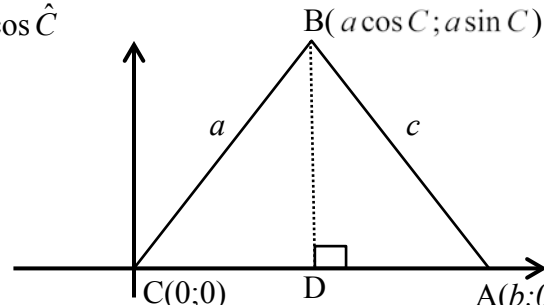
5.1.1	$x^2 + y^2 = r^2$ $(-8)^2 + (t)^2 = 17^2$ $t^2 = 225$ $t = -15$	✓ subst in pyth ✓ answer/antw (2)
5.1.2(a)	$\cos(-\theta)$ $= \cos \theta$ $= \frac{-8}{17}$	✓ $\cos \theta$ ✓ answer/antw (2)
5.1.2(b)	$1 - \sin \theta = 1 - \frac{-15}{17}$ $= \frac{17}{17} + \frac{15}{17}$ $= \frac{32}{17}$	✓ subst ✓ answer/antw (2)
5.2.1	$\tan 17^\circ = \frac{a}{\sqrt{1-a^2}}$ 	✓ sketch ✓ $\sqrt{1-a^2}$ ✓ answer/antw (3)
5.2.2	$\sin 107^\circ$ $= \sin(90^\circ + 17^\circ)$ $= \cos 17^\circ$ $= \sqrt{1-a^2}$ OR/OF $\sin 107^\circ$ $= \sin(180^\circ - 73^\circ)$ $= \sin 73^\circ$ $= \sqrt{1-a^2}$	✓ $\cos 17^\circ$ ✓ $\sqrt{1-a^2}$ (2) ✓ $\sin 73^\circ$ ✓ $\sqrt{1-a^2}$ (2)

5.2.3	$\cos^2 253^\circ + \sin^2 557^\circ$ $= (-\cos 73^\circ)^2 + (-\sin 17^\circ)^2$ $= (-a)^2 + (-a)^2$ $= 2a^2$	✓ $\cos^2 73^\circ$ ✓ $\sin^2 17^\circ$ ✓ subst of ratios ✓ answer/antw (4)
5.3	$\frac{\cos(180^\circ + 45^\circ)\sin(180^\circ - 45^\circ) + \sin(360^\circ - 30^\circ)}{\tan(180^\circ + 45^\circ)}$ $= \frac{(-\cos 45^\circ)(\sin 45^\circ) - \sin 30^\circ}{\tan 45^\circ}$ $= \frac{\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}}{1}$ $= -1$	✓ $-\cos 45^\circ$ ✓ $\sin 45^\circ$ ✓ $-\sin 30^\circ$ ✓ $\tan 45^\circ$ ✓ numerator/teller ✓ answer/antw Answer only: 1 mark out of 6 (6)
5.4	$RHS = \frac{-1}{\tan^2 x \cdot \cos^2 x}$ $= \frac{-1}{\frac{\sin^2 x}{\cos^2 x} \times \cos^2 x}$ $= \frac{-1}{\sin^2 x}$ $= \frac{-1}{1 - \cos^2 x}$ $= \frac{1}{\cos^2 x - 1}$ $= \frac{1}{(\cos x + 1)(\cos x - 1)}$ $= LHS$ OR/OF	✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ simplification/vereenv. ✓ identity ✓ factors (4)

QUESTION/VRAAG 6

6.1	$b = 30^\circ$	✓ answer/antw (1)
6.2	360°	✓ answer/antw (1)
6.3	$f(x) = g(x)$ $x = -150^\circ$ $x = 30^\circ$	✓ $x = -150^\circ$ ✓ $x = 30^\circ$ (2)
6.4	$\sin(90^\circ - x) > g(x)$ $\cos x > g(x)$ $f(x) > g(x)$ $x \in (-150^\circ; 30^\circ)$ or $-150^\circ < x < 30^\circ$	✓ $\cos x$ ✓ end points/eindpnte ✓ notation/notasie (3)
6.5	Range: $y \in [2; 4]$ or $2 \leq y \leq 4$	✓ end points/eindpnte ✓ notation/notasie (2)
		[9]

QUESTION/VRAAG 7

7.1	$AB^2 = AD^2 + BD^2$ [pythagoras] $c^2 = (b - a \cos \hat{C})^2 + (a \sin \hat{C})^2$ $= b^2 - 2ab \cos \hat{C} + a^2 \cos^2 \hat{C} + a^2 \sin^2 \hat{C}$ $= b^2 - 2ab \cos \hat{C} + a^2 (\cos^2 \hat{C} + \sin^2 \hat{C})$ $= a^2 + b^2 - 2ab \cos \hat{C}$ 	✓ $B(a \cos C; a \sin C)$ ✓ $A(b; 0)$ ✓ distance formula/ afstbdfor. ✓ expansion/ontwikk. ✓ common factor ✓ square identity (6)
7.1.2	$c^2 = a^2 + b^2 - 2ab \cos \hat{C}$ $\therefore \cos \hat{C} = \frac{a^2 + b^2 - c^2}{2ab}$ $RHS = \frac{(a+b)^2 - c^2}{2ab}$ $= \frac{a^2 + 2ab + b^2 - c^2}{2ab}$ $= \frac{a^2 + b^2 - c^2}{2ab} + \frac{2ab}{2ab}$ $= \cos \hat{C} + 1$ $= LHS$ OR/OF $c^2 = a^2 + b^2 - 2ab \cos C$ $2ab \cos C = a^2 + b^2 - c^2$ $2ab + 2ab \cos C = a^2 + 2ab + b^2 - c^2$ $2ab(1 + \cos C) = (a+b)^2 - c^2$ $1 + \cos C = \frac{(a+b)^2 - c^2}{2ab}$	✓ making cos C subject of form ✓ simplifying ✓ expansion/ontwikk. ✓ subst cos C (4) ✓ making $2ab \cos C$ the subject of the formula ✓ adding 2ab on both sides of equation ✓ common factor ✓ factorise the trinomial/drieterm (4)

7.2.1	In $\triangle ABD$ $\frac{BD}{\sin 109,6^\circ} = \frac{90,52}{\sin 31,23^\circ}$ $BD = \frac{90,52 \times \sin 109,16^\circ}{\sin 31,23^\circ}$ $= 164,92 \text{ m}$	✓ sine rule/<i>sinusreël</i> ✓ subst ✓ answer/<i>antw.</i> (3)
7.2.2	$CD^2 = 164,92^2 + 235^2 - 2 \times 164,92 \times 235 \times \cos 48,88^\circ$ $CD^2 = 31448,4874$ $CD = 177,34 \text{ m}$	✓ cos rule/<i>cosinusreël</i> ✓ subst ✓ answer/<i>antw.</i> (3)
		[16]

QUESTION/VRAAG 8

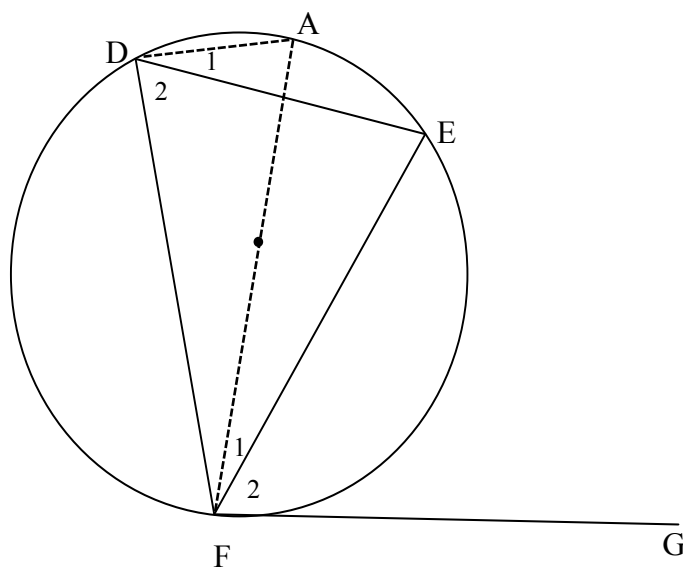
8.1	$\tan 35,5^\circ = \frac{0,5}{AB}$ $AB = \frac{0,5}{\tan 35,5^\circ}$ $= 0,7 \text{ m}$	✓ subst ✓ answer/antw (2)
8.2	Volume of cone $= \frac{1}{3} \times \pi (0,5)^2 \times 0,7$ $= 0,18 \text{ m}^3$ Volume of a cylinder $= \pi (0,5)^2 \times 1,1$ $= 0,86 \text{ m}^3$ $\frac{3}{4}$ of volume $= \frac{3}{4} \times (0,18 + 0,86)$ $= \frac{3}{4} \times (1,04) \text{ m}^3$ $= 0,78 \text{ m}^3$ <i>Time taken by pump</i> $= \frac{0,78 \text{ m}^3}{0,52 \text{ m}^3/\text{h}}$ $= 1,5 \text{ hours}$	✓ V of cone/keël ✓ V of cylinder ✓ 0,78 m ³ ✓ answer/antw (4)
		[6]

QUESTION/VRAAG 9

9.1	Equal to twice the angle subtended by the arc at the circumference	✓ ✓ answer/antw (2)
9.2.1	$\hat{R} = 30$ [$\angle$ at centre = $2 \times \angle$ at circumference]	✓ S ✓ R (2)
9.2.2	$\hat{NST} = 30^\circ$ [equal chords subtends equal angles]	✓ S ✓ R (2)
		[6]

QUESTION/VRAAG 11

11.1	$SP = SR$ [tangents from the same point] $\hat{PRS} = 42,83^\circ$ [$\angle$'s between equal sides] $\hat{ORS} = 90^\circ$ [tan $\perp$ rad] $\hat{ORN} = 90^\circ - 42,83^\circ$ $= 47,17^\circ$ $\hat{NOR} = 90^\circ - 47,17^\circ$ [sum $\angle$'s of Δ] $= 42,83^\circ$	✓ S ✓ S ✓ R ✓ $\hat{ORN}$ ✓ answer/antw (5)
11.2	<i>Let $OR = x$</i> $OS = x + 9$ $\hat{ORS} = 90^\circ$ [tan $\perp$ rad] $x^2 + 15^2 = (x + 9)^2$ [Pythagoras] $x^2 + 225 = x^2 + 18x + 81$ $18x = 144$ $x = 8 \text{ units}$ <i>radius = 8 units</i>	✓ S ✓ S ✓ Using Pythagoras ✓ answer/antw (4)
		[9]

QUESTION/VRAAG 12

12.1	Construction: Draw diameter AOF. Join A to D. $\hat{F}_1 + \hat{F}_2 = 90^\circ$ [tan $\perp$ diameter] $\hat{D}_1 = \hat{F}_1$ [$\angle$'s in the same segment] $\hat{D}_1 + \hat{D}_2 = 90^\circ$ [$\angle$ in a semi circle] $\therefore \hat{F}_2 = \hat{D}_2$ $E\hat{F}G = F\hat{D}E$	✓ Constr ✓ S ✓ R ✓ S ✓ R (5)
12.2.1	$\hat{B}\hat{A}C = 90^\circ$ [$\angle$ in a semi circle] $\hat{E}_2 = 90^\circ$ [line drawn from centre to midpont of chord] $\therefore \hat{B}\hat{A}C = \hat{E}_2$ $BA \parallel OD$ [corresp. $\angle$'s are equal] OR/OF $\hat{B}\hat{A}C = 90^\circ$ [$\angle$ in a semi circle] $\hat{E}_4 = 90^\circ$ [Line from centre to midpo int of chord] $\hat{B}\hat{A}C = \hat{E}_4$ $\Rightarrow BA \parallel OD$ [Alt $\angle$'s are equal]	✓ S / R ✓ S ✓ R ✓ R (4) ✓ S / R ✓ S ✓ R ✓ R

